Gala Cercetării Româneşti 2024

ALEXANDRU KRISTÁLY
Babeş-Bolyai University, Cluj-Napoca
CATEGORY: MATHEMATICS/INDIVIDUAL

1. Candidate

- Family name: Kristály
- Given name: Alexandru
- PhD title: 2003
- Position: Full Professor
- Institution: Babeş–Bolyai University, Cluj-Napoca
- Mobil phone:
- E-mail address:
- Web-page: /

2. Ediția "Gala Cercetării Românești": 2024.

3. Prize and category (individual or team): Mathematics, Individual.

4. Coordinator, if any: not applicable.

5. Team members, if any: not applicable.

6. Description of the most significant scientific achievements in the last 5 years

(max. 4 pages, A4 format, 12 point font): for evaluation of items C1 & C2 from Annex 3.

I am working in *Calculus of Variations, Optimal Mass Transport, PDEs* and *Geometric Analysis*, four closely related domains in modern mathematics. To underline the depth and relevance of these domains, we note that in the last 15 years two Fields Medals have been awarded to C. Villani (2010) and A. Figalli (2018), mostly for their results in Optimal Mass Transport and its applications, while in 2024, S. Brendle will receive the Breakthrough Prize in Mathematics for his contribution to geometric inequalities.

My interest/results are strongly related to the works of the aforementioned influential mathematicians, the mathematical challenges I am usually facing arise from real-life problems formulated in terms of some geometric/functional inequalities. In the last 5 years I published several key results in the theory of sharp geometric/functional inequalities, solving – among others – Lord Rayleigh's conjecture for curved clamped plates and further open problems; all these achievements are published in top mathematical journals, such as Geometric and Functional Analysis (12)[1], Journal de Mathématiques Pures et Appliquées (14), Calculus of Variations and Partial Differential Equations (20), Proceedings of the London Mathematical Society (26), Advances in Mathematics (29), Mathematische Annalen (38), Transactions of the American Mathematical Society (45), etc.

[1] In order to point out the quality of *publications* and *citations*, the red numbers in parenthesis after the journals denote the position of the given journal in the AIS list among the 330 journals from *Mathematics* or 268 journals from *Mathematics, Applied*, last edition (2023): https://uefiscdi.gov.ro/scientometrie-baze-de-date

In the sequel, I will present three of my most significant scientific achievements from the last 5 years, highlighting their concrete impacts.

I. Lord Rayleigh's conjecture for clamped plates. This class of results is probably the most impressive achievements from my carrier as a mathematical researcher. The initial conjecture – an isoperimetric problem arising from mathematical physics, – has been formulated in 1877 by Lord Rayleigh, who claimed that the disc has the minimal principal frequency (or, fundamental tone) among clamped plates with a given area. The conjecture has been solved in 1995 for 2- and 3-dimensional Euclidean spaces independently by N. Nadirashvili (*Arch. Rational Mech. Anal.*) and M. Ashbaugh and R. Benguria (*Duke Math. J.*). However, in spite of several attempts – confirmed by the above experts, – nothing has been done in the *non-flat* setting due to its involved character.

In 2020, in Advances in Mathematics (29), I proved that the fundamental tone for *small* clamped plates of low-dimensional Cartan–Hadamard manifolds (with sectional curvature bounded above by $-\kappa < 0$) are not less than the corresponding fundamental tone of a geodesic ball of the same volume in the hyperbolic space with curvature $-\kappa$. My approach requires the validity of the *Cartan–Hadamard conjecture* and peculiar properties of the Gaussian hypergeometric functions, both valid only in dimensions 2 and 3.

The icing on the cake was when I solved *Lord Rayleigh's conjecture on positively curved spaces not only in low-dimensional Riemannian manifolds with positive Ricci curvarure, but also in high-dimensions for enough large clamped domains*; the 56-page paper is published in 2022 in the top journal Geometric and Functional Analysis (12). The proof relies on an Ashbaugh–Benguria–Talenti-type nodal-decomposition, on the Lévy–Gromov isoperimetric inequality, on fine properties of hypergeometric functions and on sharp spectral gap estimates of fundamental tones for small and large clamped spherical caps.

Impact. Considered as a breakthrough achievement by the mathematical community, the result on Riemannian manifolds with positive Ricci curvature is the *first affirmative answer to the longstanding Lord Rayleigh's conjecture concerning clamped plates in high-dimensions.* The impact of these results can be measured in the number of lectures that I have been invited to give at various events, the most significant of which were on 15 May 2023 in the Partial Differential Equations Seminar of the University of Oxford, UK, and the online presentation during the Special Session at the 2022 Spring Sectional Meeting of the AMS, Purdue University, USA. In addition, recent preprints by R. Leylekian cited my papers by providing restrictive conditions (in the Euclidean setting) that yield Lord Rayleigh's conjecture.

II. Sub-riemannian geometric inequalities. In the late 2000s, J. Lott and C. Villani (*Ann. Math.*) and K.-T. Sturm (*Acta Math.*) introduced independently the famous $\mathsf{CD}(K,N)$*-condition* (CD=Curvature-Dimension), which became the starting point of the theory of synthetic geometry of metric measure spaces having Ricci curvature bounded from below and dimension controlled from above. This is nowadays called the *Lott–Sturm–Villani's theory*; within this theory, the most important geometric inequalities were given, such as the Brunn–Minkowski, Borell–Brascamp–Lieb or Prékopa–Leindler inequalities, which are important in certain entropy-based problems in mathematical physics.

The community of geometric analysts tacitly accepted the view that – contrary to the framework of $\mathsf{CD}(K,N)$-spaces of Lott–Sturm–Villani, – there are *no* similar geometric inequalities on *singular/sub-riemannian spaces*, e.g. on *Heisenberg groups.* From application point of view, one can observe that the orbits of taking-off aircrafts – which are certain 'spirals' – are precisely the geodesic segments (shortest paths) in this sub-riemannian world, that cannot be modeled by the standard Euclidean/Riemannian geometry.

In spite of this view, in 2018-2019 I published two joint papers with Z. Balogh (Bern, Switzerland) and K. Sipos (Bern) in Calculus of Variations and Partial Differential Equations (20) and Journal of Functional Analysis (40), establishing unexpected geometric inequalities in Heisenberg groups with *sub-riemannian distortion coefficients*. Our results include a sub-Riemannian version of the celebrated CD-condition of Lott–Sturm–Villani and a Borell–Brascamp–Lieb inequality akin to the one obtained in 2001 by D. Cordero-Erausquin, R.J. McCann and M. Schmuckenschläger (*Invent. Math.*). The proof is based on *Optimal Mass Transportation* on sub-Riemannian manifolds, developed in 2010 by A. Figalli and L. Rifford (*Geom. Funct. Anal.*), and shows the genuine difference between Riemannian and sub-riemannian geometries.

Impact & citations. The impact of our results is multiple. First, from application point of view, the geometric inequalities we stated describe real-world situations of certain optimization problems, where the usual models/approaches fail. For instance, the orbits of airplanes (or the deterministic movement of a car) can be realistically described within the geometry of sub-riemannian manifolds, contrary to the classical euclidean setting.

Second, from pure scientific point of view, our results have a huge impact, being extended in 2019 to generic sub-riemannian structures by D. Barilari and L. Rizzi in Invent. Math. (6). Moreover, based on the papers of Balogh–Kristály–Sipos and Barilari–Rizzi, in 2019 the Fields medalist C. Villani formulated in Astérisque (27) the problem of „*grand unification*" of geometric inequalities on *curved spaces*.

Beside the aforementioned works by Barilari–Rizzi and Villani, the papers by Balogh–Kristály–Sipos emerged important works, being further cited – among others – by E. Milman in Comm. Pure Appl. Math. (2), T. Mietton and L. Rizzi in Geom. Funct. Anal. (12), M. Ritoré and J.Y. Nicolás in Adv. Math. (29), D. Barilari and L. Rizzi in Math. Ann. (38), etc.

In a very recent study of D. Barilari (Padova), A. Mondino (Oxford) and L. Rizzi (Grenoble), – where Villani's "*grand unification*" problem is solved – the authors use the notion of "Balogh–Kristály–Sipos' theory".

In addition, I have been invited to various important conferences in this topic, see e.g. *23th Rolf Nevanlinna Colloquium* (ETH Zürich, 2017), XIV-ème Colloque Franco-Roumain de Mathématiques Appliquées (Université de Bordeaux, France, 2018), Workshop "Geometry and Probability" (Fukuoka, Japan, 2019), 30th European Conference on Operational Research (Dublin, Ireland, 2019), etc.

III. Sharp isoperimetric inequalities. Isoperimetric inequalities are probably the most celebrated problems in Geometric Analysis, due to their simple formulations and applicability. Their study is still very active, requiring complex mathematical arguments/tools, see e.g. the works of the Fields medalist A. Figalli. Just to emphasize the hardness of the problem, it is worth to recall the Cartan–Hadamard conjecture, formulated by M. Gromov and Th. Aubin in the eighties; they claimed that formally the same sharp isoperimetric inequality is valid on Cartan–Hadamard manifolds as in Euclidean spaces. Until now, the conjecture is verified only in low-dimensions (2, 3 and 4).

Trying to solve the Cartan–Hadamard conjecture in high-dimensions by the *Optimal Mass Transport* theory – without success till now – Z. Balogh (University of Bern) and myself observed that the *sharp isoperimetric inequality holds on* $\mathsf{CD}(0, N)$ *metric measure spaces*, i.e., spaces which are curved in the sense of Lott–Sturm–Villani; the paper is published in 2023 in the famous journal Mathematische Annalen (38). It is worth to emphasize that this isoperimetric inequality contains the so-called *Asymptotic Volume Ratio*, – a notion used by G. Perelman in Riemannian geometry, – which is a geometric invariant providing crucial information on the topological structure of the space. Moreover, the main result of S. Brendle published in 2023 in Comm. Pure Appl. Math. (2) – which is proved by a PDE-technique on Riemannian manifolds – appears as a consequence of our result.

Even more exciting, by this sharp isoperimetric inequality, we proved in the same paper *sharp Sobolev inequalities on Riemannian manifolds having nonnegative Ricci curvature*, answering an open problem from the AB-program, initiated by Th. Aubin in the seventies. In particular, the latter result provides a one-line proof of the famous rigidity theorem of C. Xia and M.P. do Carmo from 2004 published in *Compos. Math.*

Impact & citations. Although the result is very recent (2023), it has a huge impact, being cited by leading mathematical figures as E. Brué (Princeton), D. Semola (Oxford), M. Fogagnolo and A. Malchiodi (Pisa), etc., in papers published in leading journals as Math. Annalen (38), Calc. Var. and PDEs (20), etc. These papers deal with the characterization of the equality case and stability in our sharp isoperimetric inequality. Another related citation is the recent work published in Ann. Inst. H. Poincaré Anal. Non Linéaire (16) to our paper from Advances in Mathematics (29) by the Fields medalist A. Figalli.

In addition, due to these results, I was invited by R. McCann in November 2022 to the *Fields Institute*, Toronto, Canada. By meantime, I had several talks on this subject, see e.g. International Conference on Variational Analysis and Optimization with Applications, Aligarh Muslim University, India (2023), AMS special session "Geometric and Functional Inequalities and Applications to PDEs" (2022), etc.

Beyond the scientific impact, the *Optimal Mass Transport* arguments in our proofs could be efficiently applied also in real life. Indeed, the idea is to transport a mass from one place to another by using the least energy/work w.r.t. a certain cost function. This theory has spectacular applications in Economics (allocation of resources), meteorology (migration of clouds), chemistry (movement of particles), etc. Our results put all these problems into a *curved* setting, where the geometry of the ambient space has a key influence.

7. **Narrative Curriculum Vitae** (focusing mostly to the last 5 years): for evaluation of items C1 - C5 from Annex 3.

Short biosketch. I completed my studies at the Faculty of Mathematics of the Babeş-Bolyai University, Cluj-Napoca, Romania (1993-1998). I defended my first PhD Thesis (in 2003) in analysis under the supervision of W.W. Breckner at Babeş-Bolyai University, the second one (in 2005) in Riemannian-Finsler geometry under the supervision of L. Kozma at the University of Debrecen (Hungary) and the third one (in 2010) in mathematical economics under the supervision of G. Moroşanu at Central European University, Budapest. I also received (in 2019) the title of Doctor in Science from the Hungarian Academy of Sciences, Budapest. At the present, I am a Full Professor at Babeş-Bolyai University, Cluj-Napoca, and a Senior Research Fellow of the Óbuda University, Budapest, Hungary.

Research interest. I am working in the interface of Calculus of Variations, Geometric Analysis, PDEs and Optimal Mass Transportation, investigating various nonlinear phenomena arising from mathematical physics and (synthetic or differential) geometry where the curvature of the ambient space has a decisive role.

Most relevant scientific results & citations (last 5 years): for C1 and C2 from Annex 3. A detailed description can be found in item **6**; here, nutshell descriptions are provided.

(a) I solved Lord Rayleigh's conjecture for fundamental tones of clamped plates both on negatively and positively curved spaces, by showing the peculiar curvature and dimension dependence of this problem: in 2020, in negatively curved spaces, the conjecture is confirmed for 2 and 3 dimensional enough small domains in Advances in Mathematics (29), while in 2022 for enough large domains in high-dimensional positively curved spaces in the top journal Geometric and Functional Analysis (12); the latter achievement provides the first answer when the conjecture is confirmed in high-dimensions. These results seem to be the most relevant achievements in my carrier.

(b) Jointly with Z. Balogh and K. Sipos, we proved in 2018-2019 sharp geometric inequalities on Heisenberg/Carnot groups, refuting the tacitly accepted view that no such inequalities can be stated on singular spaces, see Calculus of Variations and Partial Differential Equations (20) and Journal of Functional Analysis (40). Our results led the Fields medalist C. Villani to propose the „grand unification" of geometric inequalities on non-Euclidean structures (including Riemannian, Finsler and sub-Riemannian manifolds), which provided a deep source of inspiration for researchers working in leading mathematical centers (Princeton, Oxford, Pisa, Paris , SISSA, etc.). These researchers cited our results in leading journals as Invent. Math. (6), Comm. Pure Appl. Math. (2), Astérisque (27), Geom. Funct. Anal. (12), Adv. Math. (29), Math. Ann. (38), J. Funct. Anal. (40), just to call a few of them. In a very recent study of D. Barilari (Padova), A. Mondino (Oxford) and L. Rizzi (Grenoble), – where Villani's "*grand unification*" problem is solved – the authors use the formulation of "Balogh–Kristály–Sipos' theory".

(c) In 2023, in a joint work with Z. Balogh in Mathematische Annalen (38) we proved the sharp isoperimetric inequality on metric spaces that are curved in the sense of Lott–Sturm–Villani. Moreover, we solved an open question of Th. Aubin from the seventies concerning the sharp Sobolev inequalities on Riemannian manifolds with nonnegative Ricci curvature, and it turned out that the best constant contains the *Asymptotic Volume Ratio* of the manifold, a geometric invariant introduced/used by G. Perelman. This result is already cited in more than 30 papers, among others in Math. Ann. (38), Calc. Var. and PDEs (20), etc, by leading mathematicians from Oxford, Princeton, Paris, Pisa, etc.

(d) Spectacular results in the theory of sharp Sobolev inequalities on geometric objects were obtained jointly with W. Zhao in 2022 in Journal de Mathématiques Pures et Appliquées (14), with Z. Balogh and C. Gutiérrez in 2021 in Proceedings of the London Mathematical Society (26), and with L. Huang and W. Zhao in 2020 in Transactions of the American Mathematical Society (45), where we applied *Optimal Mass Transport* theory with topological rigidity arguments. These results are cited in papers published by various authors in Calc. Var. and PDEs (20), Proc. Amer. Math. Soc. (87), etc.

Prestige, international recognition & prizes (last 5 years): for C4 from Annex 3.

(i) *Level of appreciation*: During the last 5 years I was invited to various research centers, seminar talks or plenary lectures: Institute of Mathematics of the University of Oxford (2023), Fields Institute, University of Toronto, Canada (2022), Institute of Mathematics of the Universität Bern (2019-2023), Alfréd Rényi Institute of Mathematics, Hungarian Academy of Sciences, Budapest (2019), University of Fukuoka, Japan (2019), etc.

(ii) *Doctor in Science* (DSc): Hungarian Academy of Sciences, Budapest, 2019. Title of thesis: Sharp functional inequalities and elliptic problems on non-euclidean structures.

(iii) *Editorial activity*: I am an Editor of *Journal of Optimization Theory and its Applications* (Springer), a Q2 journal w.r.t. AIS, as well as of *Analele Universităţii din Timişoara, Seria Matematica* (De Gruyter), *Acta Universitatis Sapientiae, Mathematica* (De Gruyter), and *Studia Universitatis Babeş-Bolyai, Mathematica.*

(iv) *Books/monographs published in prestigious editors*:

- Costea N, Kristály A, Varga C, Variational and Monotonicity Methods in Nonsmooth Analysis, Frontiers in Mathematics, Birkhäuser/Springer, 2021.

- Kristály A, Rădulescu V, Varga C, Variational Principles in Mathematical Physics, Geometry, and Economics, Encyclopedia of Mathematics and its Applications, No. 136, Cambridge University Press, Cambridge, UK, 2010.

The latter monograph has 209 citations in the MathSciNet database.

(v) *Prizes*: In 2020, I have been awarded with the *Ad Astra Award* in Mathematics, Bucharest. In 2021, I got the *Arany János Prize for Outstanding Scientific Achievements* by the Hungarian Academy of Sciences, Budapest, Hungary.

Earlier, in 2014, I received the *Spiru Haret Award* from the Romanian Academy, Bucharest, and in 2013, the *Bolyai Plaque* by the Hungarian Academy of Sciences, Budapest.

Organizational capacities & supervision: for C5 from Annex 3.

(i) *Supervision activity.* Until this moment, I was/am the supervisor of 7 PhD Students:

- Defended (4): S. Nagy (2015), C. Farkas (2018), O. Vas (2021), A. Mester (2023);
- In progress (3): K. Szilák, B. Oltean-P., and S. Kajántó.

(ii) *Scientific seminar.* Within the research grant Eigenvalues on curved spaces, we organized in Cluj-Napoca period reading/research (online/offline) seminars in the topic of Geometric Analysis, discussing research papers and ideas in the theory of geometric/functional inequalities and related topics.

(iii) *Effect of supervision.* My PhD Students already published jointly or alone remarkable research papers (in Q1 and Q2 journals), see e.g. in Calc. Var. and PDEs (20), J. Differential Equations (51), Comm. Cont. Math. (64), Adv. Nonlinear Stud. (73), etc.

(iii) *Conference organizations* (last 1 year).

- Member of the Scientific Committee: Colloquium on Finsler Geometry and its Applications, University of Debrecen, Hungary, 12-16 June 2023.
- Section organizer: 20th EUROpt Workshop: continuous optimization working group is coming home, organization of the Stream "*Optimization on manifolds*", Corvinus University of Budapest, Hungary, 23-25 August 2023.
- Section organizer: *SACI 2023*, Special Session on Applied Mathematics: Nonlinear Phenomena, Timişoara, Romania, 23-25 May 2023.

8. List of publications.

Full information about my publications can be found at the personal web-page:
https://alexandrukristaly.wordpress.com/
or at the MathSciNet database of the American Mathematical Society
https://mathscinet.ams.org/mathscinet/.

Beside the two monographs mentioned above in **7.**/(iv), edited by Birkhäuser/Springer in 2021 and Cambridge University Press in 2010, I published research papers in top mathematical journals as Geometric and Functional Analysis (12), Journal de Mathématiques Pures et Appliquées (14), Calculus of Variations and Partial Differential Equations (20), Proceedings of the London Mathematical Society (26), Advances in Mathematics (29), Mathematische Annalen (38), Journal of Functional Analysis (40), Transactions of the American Mathematical Society (45), J. Differential Equations (51) etc.

The list of my research papers in the last 5 years can be found in the following table, where Q means the quadrille of journals in the AIS list, while n stands for the number of authors:

No.	**Research papers from the last 5 years (A. Kristály)**	Q	AIS	n	AIS/n
1	Kristály A, Mester Á, Mezei I. I, *Sharp Morrey-Sobolev inequalities and eigenvalue problems on Riemannian-Finsler manifolds with nonnegative Ricci curvature*, COMM. CONTEMP. MATH. 25 (2023), no. 10, Paper No. 2250063.	Q1	1,227	3	0,409
2	Balogh Z, Kristály A, *Sharp isoperimetric and Sobolev inequalities in spaces with nonnegative Ricci curvature*, MATH. ANNALEN, 385 (2023), no. 3-4, 1747–1773.	Q1	1,671	2	0,835
3	Kristály A, *Lord Rayleigh's conjecture for vibrating clamped plates in positively curved spaces*, GEOM. FUNCT. ANAL. (GAFA), 32 (2022) 881–937	Q1	3,351	1	3,351
4	Kristály A, *New features of the first eigenvalue on negatively curved spaces*, ADV. CALC. VAR., 15 (2022), no. 3, 475–495.	Q1	1,344	1	1,344
5	Kristály A, Zhao W, *On the geometry of irreversible metric-measure spaces: convergence, stability and analytic aspects*, J MATH PURES APPL (Liouville Journal), 158 (2022), 216–292.	Q1	2,066	2	1,033
6	Farkas C, Kristály A, Mester Á, *Compact Sobolev embeddings on non-compact manifolds via orbit expansions of isometry groups*, CALCULUS OF VARIATIONS & PDEs, (2021) 60:128.	Q1	1,792	3	0,597
7	Balogh Z, Gutiérrez E. C, Kristály A, *Sobolev inequalities with jointly concave weights on convex cones*, PROC. LOND. MATH. SOC., 122 (2021), no. 4, 537–568.	Q1	2,096	3	0,698
8	Huang L, Kristály A, Zhao W, *Sharp uncertainty principles on general Finsler manifolds*, TRANS. AMER. MATH. SOC., 373 (2020), no. 11, 8127–8161.	Q1	1,554	3	0,518
9	Kristály A, *Fundamental tones of clamped plates in nonpositively curved spaces*, ADV. MATH. 367 (2020), 107113, p. 39.	Q1	1,965	1	1,965
10	Kristály A, Mezei I, Szilák K, *Differential inclusions involving oscillatory terms*, NONLINEAR ANAL. 197(2020) 111834, 21.	Q1	0,964	3	0,321
11	Balogh Z, Kristály A, Sipos K, *Jacobian determinant inequality on corank 1 Carnot groups with applications*, J. FUNCT. ANAL., 277 (2019), no. 12, 108293, p. 36.	Q1	1,635	3	0,545
12	Kristály A, *New geometric aspects of Moser-Trudinger inequalities on Riemannian manifolds: the non-compact case*, J. FUNCT. ANAL. 276 (2019), no. 8, 2359–2396.	Q1	1,635	1	1,635
13	Kristály A, Szakál A, *Interpolation between Brezis-Vázquez and Poincaré inequalities on nonnegatively curved spaces: sharpness and rigidities*, J. DIFFERENTIAL EQUATIONS, 266 (2019), no. 10, 6621-6646.	Q1	1,416	2	0,708
14	Kristály A, Mezei I. I, Szilák K, *Elliptic differential inclusions on non-compact Riemannian manifolds*, NONLINEAR ANALYSIS-REAL WORLD APPL. 69 (2023), 103740.	Q2	0,887	3	0,295
15	Kristály A, Shen Z, Yuan L, Zhao W, *Nonlinear spectrums of Finsler manifolds*, MATH. Z. 300 (2022), 81–123.	Q2	1,049	4	0,262
16	Kajántó S, Kristály A, *Unexpected Behaviour of Flag and S-Curvatures on the Interpolated Poincaré Metric*, J. GEOM. ANAL. 31 (2021), 10246–10262.	Q2	0,854	2	0,427
17	Kristály A, *Nodal solutions for the fractional Yamabe problem on Heisenberg groups*, PROC. ROY. SOC. EDINBURGH SECT. A, 150 (2020), no. 2, 771–788.	Q2	0,819	1	0,819
					15,764

9. Attracting funds, research grants & collaborations (last 5 years): for C3 from Annex 3.

(i) *Program coordinator* (Senior category). Title of the grant: Eigenvalues on curved spaces, 2021-2023, PN-III-P4-ID-PCE-2020-1001, CNCS, Bucharest, Romania.

Budget: 1.198.032 RON (≈ 240.725 Euro).

Four members involved: C. Farkas, S. Kajántó, I.I. Mezei and C. Varga (who passed away in August 2021).

(ii) *Program coordinator* (Senior category). Title of the grant: *Functional inequalities and elliptic PDEs: the influence of curvature*, 2018-2022, National Research, Development and Innovation Fund of Hungary, K_18, No. 127926.

Budget: 25.000 Euro.

I was a Research Fellow at the University of Bern in October and November 2023, June 2022, October 2020, etc.

During the last period I collaborated with the following mathematicians (which are usually reflected in joint publications): Z. Balogh (Bern, Switzerland), C. Gutiérrez (Temple University, USA), L. Huang (Tianjin, China), A. Mondino (Oxford, UK), Z. Shen (Indianapolis, USA), A. Szakál (Budapest, Hungary), F. Tripaldi (Bologna, Italy), W. Zhao (Shanghai, China), and other Romanian colleagues.

Cluj-Napoca, 28 January, 2024 Alexandru Kristály